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GPS L1-L2 Bias Determination

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12 January 1993

Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LEXINGTON, MASSACHUSETTS



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GPS L1-L2 BIAS DETERMINATION

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TECHNICAL REPORT 971

12 JANUARY 1993

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ABSTRACT

The GPS satellite L1-L2 bias T_{gd} is determined using data from a TI-4100 receiver operating at Millstone Hill in Westford, MA. Pseudorange L1-L2 differences are computed. This difference is a measure of the total electron content (TEC) path delay along the line of sight plus the L1-L2 satellite and receiver bias. The difference of two measurements at the same time and satellite position cancels the TEC path delay and possible receiver bias and is a measure of the difference in satellite bias of the two satellites; i.e., $Sv(a)L1-L2 - Sv(b)L1-L2$. Such direct measurements are not often available. Therefore, measurements at the same time are converted to "zenith" TEC values using a mapping function. These differences are used in a least squares solution to determine the difference in bias between GPS satellites. Individual bias differences are obtained with an accuracy of 0.1 ns (0.285 TEC units) or better. By choosing one satellite as the reference, the individual satellite biases can also be obtained. The stability of each satellite bias is also estimated by examining a time series of the differences.

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1. INTRODUCTION

The Global Positioning System (GPS) is based on measuring the travel time Δt of a signal from the satellite to a receiver. The method used to obtain Δt is not relevant here. The travel time can be used to obtain the distance r to the satellite using $r = c\Delta t$, where c is the speed of light. This simple use of the travel time is compromised by media effects that increase the travel time compared to that in vacuo, i.e., ionospheric and tropospheric refraction. Only the ionospheric path delay is addressed here.

The index of refraction in the ionosphere has sufficient dispersion at L-band frequencies to allow measurement of the integrated ionospheric path delay. To deal with ionospheric path delay, the GPS system provides travel time, i.e., ranging data, at two frequencies. These are called the L1 and L2 frequencies: 1575.42 and 1227.60 MHz, respectively. This allows direct measurement of the ionospheric path delay along the path for each range measurement. A simultaneous measurement from a number of GPS satellites can also be used for synoptic monitoring of the ionosphere in the neighborhood of the observing station [1].

It has been observed in both preflight calibration and analysis of GPS data that there is a systematic time bias between the time transmitted at L1 and L2. This time bias seems to be different for each GPS satellite, and it may vary with time. The prelaunch measurements made by Rockwell are rather noisy [2], and some biases are possibly smaller than their uncertainty. There have also been a number of attempts to determine the individual space vehicle (Sv) biases [2,3,4,5,6] which we have summarized in Table 5. The results have been inconsistent at best, and with no good agreement. This report is another contribution to the subject.

A constant L1-L2 bias would result in a constant error in the ionosphere path delay determination and the range measurement. A constant L1-L2 bias would then appear as a bias in L1, i.e., an Sv clock error. Therefore, it would not compromise analysis that was independent of clock error such as use of double differences.

All attempts to determine the Sv bias using GPS observations hinge on any possible bias, and its change, in the GPS receiver used for the data acquisition. No conclusive evidence on this point is available. There has been a relevant and extensive study [7] on more than 50 TI-4100 receivers. The basic result was that, though not all the receiver biases were small, the receiver bias was constant. In addition, the receiver at Millstone Hill has been used in a number of comparisons with other measurements of the ionospheric path delay [8,9]. These comparisons include incoherent scatter radar measurements of the ionosphere, Faraday rotation data, and Digisonde measurements. In cases where there was effectively no ionospheric path delay, the TI-4100 measured no additional time delay. Based on these two facts, an assumption is made that there is effectively no receiver bias in the following analysis.

2. ESTIMATION PROCEDURE

Each observation is $O = t_{L1} - t_{L2} = \Delta t_{TEC} + \Delta t_{bias} + \epsilon$, where t_{L1} and t_{L2} are the broadcast L1 and L2 times received at the observation time, Δt_{TEC} is the difference in propagation time for L1 and L2 caused by the ionospheric path delay, Δt_{bias} is the difference in time as broadcast by the satellite (i.e., the bias), and ϵ is the measurement error. In the following we assume that the $E\{\epsilon\}=0.0$ with a standard deviation to be determined. A nonzero mean measurement error will be absorbed in Δt_{bias} . The dominant pseudorange error is due to multipath, as evidenced by the daily (~ 4 min) repeatability of the error signature. There is no evidence that multipath is or is not a zero mean error.

The path delay at a given frequency is proportional to the total electron content (TEC) along the path. The TEC is conventionally measured in TEC units of 10^{16} electrons/m². For the L1 (1575.42 MHz) and L2 (1227.60 MHz) frequencies, one can show that $\Delta t_{TEC} = [1 - (1575.42/1227.60)^2] \times \text{TEC}/(c \times 6.159)$, where c is in m/s and Δt is in seconds. So we have, $O = T + B$, the sum of the TEC dependent path delay along the line of sight T , (expressed in seconds of path delay) and the satellite L1-L2 bias B . With two observations, O_i and O_j , along the same line of sight at the same time, the difference would be $O_i - O_j = B_i - B_j$, the difference in satellite bias. This condition is not generally available.

Observations at different positions, at the same time, can be used in the following way. Each line of sight TEC can be converted to a vertical TEC T_o at the ionosphere pierce point using a mapping function $Z(ele)$, which generally depends on elevation [1]. The line of sight TEC T is simply related to the vertical TEC T_o with $T = Z \times T_o$. Assuming a spherically stratified symmetric ionosphere, each observation has the same vertical ionosphere TEC T_o . Therefore, we have an observation of

$$\frac{O}{Z} = \frac{T}{Z} + \frac{B}{Z} = T_o + \frac{B}{Z} = o$$

By subtracting two contemporaneous measurements,

$$o_i - o_j = \frac{B_i}{Z_i} - \frac{B_j}{Z_j}$$

Many measurements could be used in a joint least squares computation of B_i and B_j . However, there are two issues here. First, the assumption of spherical symmetry is not very good for low elevations. Therefore, the analysis was restricted to elevations above 30 deg. Second, with this restriction the mapping function has a small variation, and the least squares solution is nearly singular with highly correlated errors. To avoid this drawback, the problem is reformulated in terms of the difference in bias, $y = B_i - B_j$, and one of the biases, B_j , as

$$o = \frac{y}{Z_i} + \left(\frac{1}{Z_i} - \frac{1}{Z_j} \right) B_j$$

The joint least squares solution for y and B_j is more robust for y .

With the use of contemporaneous observations at different positions, a number of satellite combinations can occur. We then seek a joint solution. This mathematical discussion is limited to three satellites. The formulation is easily extended to any number of satellites, as was done for the numerical results given in our data analysis. In every case, a single reference bias is chosen. All observed combinations can then be obtained. Consider three contemporaneous observations, o_i , o_j , and o_k , and choose B_k as the reference bias. Then the unknowns are $y_i = B_i - B_k$, $y_j = B_j - B_k$, and B_k . The observation equations are then

$$\begin{bmatrix} \frac{1}{Z_i} & 0 & \frac{1}{Z_i} - \frac{1}{Z_k} \\ 0 & \frac{1}{Z_j} & \frac{1}{Z_j} - \frac{1}{Z_k} \\ \frac{1}{Z_i} & -\frac{1}{Z_j} & \frac{1}{Z_i} - \frac{1}{Z_j} \end{bmatrix} \begin{bmatrix} y_i \\ y_j \\ B_k \end{bmatrix} = \begin{bmatrix} o_i - o_k \\ o_j - o_k \\ o_i - o_j \end{bmatrix}$$

These observation equations can be solved by the method of least squares. Inversion of the normal equations is done with a singular value decomposition, again preserving the robust solution for y_i and y_j . In the analysis described below, the reference bias B_k is also well determined.

3. DATA ANALYSIS

A GPS receiver operating at Millstone Hill routinely acquired and archived data from April 1991 through May 1992. For this period there was data from the Sv's, as given in Table 1. This table includes some general information about the satellites. The PRN number is used as the Sv identification number in the following discussion. The Space Surveillance Network number, the international designation, and the NAVSTAR number are also provided. The Block I and Block II satellites are identified, along with various designations, the launch date, and the type of on-board clock.

TABLE 1
GPS Satellites Used in Analysis

PRN	SSN	Int Des	BI	Name	Clock	Launch
2	20061	8904401	II	USA-38,NAV13	Ces.	10Jun89
3	16129	8509301	I	USA-10,NAV11		9Oct85
6	11054	7809391	I	GPS3,NAV3	Rub.	6Oct78
11	14189	8307201	I	GPS7,NAV8	Ces.	14Jul83
12	15271	8409701	I	USA-5,NAV10	Ces.	8Sep84
13	15039	8405901	I	USA-1,NAV9	Ces.	13Jun84
14	19802	8901301	II	USA-35,NAV14	Ces.	14Feb89
15	20830	9008801	II	USA-64,GPS_2-09,NAV15	Ces.	1Oct90
16	20185	8906401	II	USA-42,NAV16	Ces.	18Aug89
17	20361	8909701	II	USA-49,GPS_2-05,NAV17	Ces.	11Dec89
18	20452	9000801	II	USA-50,GPS_2-06,NAV18	Ces.	24Jan90
19	20302	8908501	II	USA-47,GPS_2-04,NAV19	Ces.	21Oct89
20	20533	9002501	II	USA-53,NAV20	Ces.	26Mar90
21	20724	9006801	II	USA-63,NAV21	Ces.	2Aug90
23	20959	9010301	II	USA-66,GPS_2-10	Ces.	26Nov90

This analysis was confined to pseudorange data. This type of data is much noisier than the carrier phase data, although some improvement in noise performance has been achieved [10]. The pseudorange data was differenced to obtain the line of sight observation O . This value was converted to an equivalent vertical observation o using the mapping function $Z(ele)$ based on a stratified spherically symmetric slab model of the ionosphere,

$$Z(ele) = \frac{\sqrt{(a+h+s)^2 - [a \cos(ele)]^2} - \sqrt{(a+h)^2 - [a \cos(ele)]^2}}{s}$$

where a is the height of the observing station from the earth center, $h = 300$ km is the height of the ionosphere slab, $s = 200$ km is the slab thickness, and ele is the elevation.

TABLE 2
Statistics Of Least Squares Solution

Month	Sigma (ns)	Num. Obs. Accepted	Num. Obs. Rejected	Num. Iter.
April 1991	3.41	49290	422	4
May 1991	3.27	28386	215	4
June 1991	3.17	23372	240	4
July 1991	3.10	18835	136	4
Aug 1991	3.14	23830	154	4
Sep 1991	3.18	37876	302	4
Oct 1991	3.28	45734	526	4
Nov 1991	3.08	28388	364	4
Dec 1991	2.77	34605	317	4
Jan 1992	3.06	31371	260	4
Feb 1992	3.37	38665	528	4
March 1992	3.34	42080	371	4
April 1992	2.95	43708	292	4
May 1992	2.75	22292	197	4

The data were taken in one calendar month blocks. Sv 17 was chosen as the reference satellite, as previous analysis suggested it had a stable, constant bias. The simultaneous solution for the Sv bias differences for 14 satellites and the bias for Sv 17 was done for each month. The least squares solution was iterated, using a 3 sigma screening criterion. The number of iterations, standard deviation, and number of observations used for each month are given in Table 2. Figures 1(a) and (b) are an example of the least squares solution. Figure 1(a) shows the selection of data by day. Figure 1(b) shows, by Sv number, the mean L1-L2 and standard deviation for the input data, the number of input measurements, the number discarded being below 30 deg elevation, and the number used in the regression analysis. Figure 1(c) shows the iteration history, the solution vector and formal uncertainty in nanoseconds, and the correlation matrix in percent. The formal uncertainty is approximately 0.1 ns. The correlation coefficients are small enough to believe that each parameter is uniquely determined from the available data. Finally, Figure 1(d) shows the statistics for the data by Sv, the mean residual, and the number of observations accepted and rejected. Note that an observation is used in multiple residual calculation, so the number used here may be greater than the corresponding number in Figure 1(b).

Get data from /gps/92apr01	Get data from /gps/92apr16
Get data from /gps/92apr02	Get data from /gps/92apr17
Get data from /gps/92apr03	Get data from /gps/92apr18
Get data from /gps/92apr04	Get data from /gps/92apr19
Get data from /gps/92apr05	Get data from /gps/92apr20
Get data from /gps/92apr06	Get data from /gps/92apr21
Get data from /gps/92apr07	Get data from /gps/92apr22
Get data from /gps/92apr08	Get data from /gps/92apr23
Get data from /gps/92apr09	Get data from /gps/92apr24
Get data from /gps/92apr10	Get data from /gps/92apr25
Get data from /gps/92apr11	Get data from /gps/92apr26
Get data from /gps/92apr12	Get data from /gps/92apr27
Get data from /gps/92apr13	Get data from /gps/92apr28
Get data from /gps/92apr14	Get data from /gps/92apr29
Get data from /gps/92apr15	Get data from /gps/92apr30

(a)

Figure 1(a). Data for Sv bias from April 1992.

		200005-05				
		$\langle L1-L2 \rangle$ (ns)	$\sigma(L1-L2)$ (ns)	n_{in}	n_{bad}	n_{used}
20061 = Sv	2	14.123	4.760	4738	932	4546
16129 = Sv	3	7.211	3.296	3506	4608	2962
11054 = Sv	6	15.034	4.266	3213	3249	3069
14189 = Sv	11	18.945	4.726	885	980	882
15271 = Sv	12	11.405	3.642	4251	3542	4121
15039 = Sv	13	13.205	4.606	4749	1734	4224
19802 = Sv	14	15.531	5.583	4069	3939	1854
20630 = Sv	15	15.462	4.378	4239	2874	3806
20185 = Sv	16	6.967	3.857	3887	1782	3512
20452 = Sv	18	17.206	4.867	3085	5310	1488
20302 = Sv	19	17.249	4.935	4641	2157	3609
20533 = Sv	20	13.866	4.951	2411	5247	926
20724 = Sv	21	8.295	4.690	3995	1355	3631
20959 = Sv	23	6.770	4.721	6242	2305	5288
21552 = Sv	24	7.426	3.617	7045	1893	5305
20361 = Sv	17	5.610	3.545	4165	3280	3884

(b)

Figure 1(b). Input data statistics of April 1992.

Iter 1 sigma = 3.563 44000

Iter 2 sigma = 3.011 43914

Iter 3 sigma = 2.956 43746

Iter 4 sigma = 2.945 43708

Sv(02-17)	(1.33 + -0.1)	E O	100	37	88	69	13	79	74	90	58	66	85	25	21	23	64	-60
Sv(03-17)	(2.59 + -0.1)	E O	37	100	34	27	14	44	31	34	56	25	33	13	17	23	54	-33
Sv(06-17)	(1.54 + -0.1)	E O	88	34	100	88	12	73	69	90	53	67	85	23	20	22	59	-56
Sv(11-17)	(5.83 + -0.2)	E O	69	27	68	100	9	56	52	70	42	69	77	18	16	17	46	-47
Sv(12-17)	(2.50 + -0.1)	E O	13	14	12	9	100	13	11	12	13	9	12	23	64	66	13	-15
Sv(13-17)	(4.26 + -0.1)	E O	79	44	73	56	13	100	70	73	68	54	69	24	20	22	77	-54
Sv(14-17)	(-5.22 + -1.4)	E-1	74	31	69	52	11	70	100	69	48	51	65	29	18	18	55	-38
Sv(15-17)	(1.45 + -0.1)	E O	90	34	90	70	12	73	69	100	54	68	87	23	20	22	60	-57
Sv(16-17)	(1.76 + -0.1)	E O	58	56	53	42	13	68	48	54	100	40	52	18	19	23	82	-53
Sv(18-17)	(1.42 + -0.2)	E O	66	25	67	69	9	54	51	68	40	100	78	17	14	16	44	-41
Sv(19-17)	(3.11 + -0.1)	E O	85	33	85	77	12	69	65	87	52	78	100	22	19	21	57	-56
Sv(20-17)	(-1.14 + -0.1)	E O	25	13	23	18	23	24	29	23	18	17	22	100	35	25	20	-20
Sv(21-17)	(-1.54 + -0.1)	E O	21	17	20	16	64	20	18	20	19	14	19	35	100	65	20	-27
Sv(23-17)	(-9.48 + -0.5)	E(-1)	23	23	22	17	66	22	18	22	23	16	21	25	65	100	23	-31
Sv(24-17)	(1.50 + -0.1)	E O	64	54	59	46	13	77	55	60	82	44	57	20	20	23	100	-53
Sv(17) ...	(-1.62 + -0.1)	E O	-60	-33	-56	-47	-15	-54	-38	-57	-53	-41	-56	-20	-27	-31	-53	100

(c)

Figure 1(c). L2 solution and statistics for Sv bias April 1992.

			200025-26	
	$\langle L1-L2 \rangle$ (ns)	$\sigma(L1-L2)$ (ns)	n_{good}	n_{bad}
20061 = Sv 2	-0.12	2.80	8622	37
18129 = Sv 3	0.06	2.89	4577	18
11054 = Sv 8	-0.03	2.88	6332	42
14189 = Sv 11	-0.24	3.21	1451	20
15271 = Sv 12	-0.06	3.08	7614	81
15039 = Sv 13	0.10	2.93	5621	30
19802 = Sv 14	-0.00	2.72	2980	7
20830 = Sv 15	-0.01	3.00	7348	43
20185 = Sv 16	-0.04	2.98	4701	31
20452 = Sv 18	0.23	2.99	1771	18
20302 = Sv 19	-0.07	2.90	6395	45
20533 = Sv 20	0.02	3.06	930	16
20724 = Sv 21	0.02	3.09	6698	71
20959 = Sv 23	0.02	2.99	9321	67
21552 = Sv 24	-0.12	3.01	6320	35
20361 = Sv 17	0.31	2.75	6755	23

(d)

Figure 1(d). Residual's statistics for April 1992.

The individual solutions for bias differences are given in Table 3. These are believed to be uncontaminated by receiver bias. Sv 17 is selected as the reference. If another Sv was chosen, the difference in bias for any two Sv's would be the same. Table 3 also contains the average for all solutions and the standard deviation of individual solutions about the average. Since these are independent observations of the bias, the formal accuracy of each bias value is the standard deviation divided by the square root of 13.

The Sv bias is given in Table 4 for each month using the monthly recovered value for Sv 17. Table 4 also contains the average for each Sv bias and the standard deviation of the individual bias about the average. Since these are independent observations of the bias, the formal accuracy of each bias value is the standard deviation divided by the square root of 13. Figures 2 through 16 show the time history for the bias of each Sv.

TABLE 3
Solution For Sv Bias Differences
(L1-L2 in nanoseconds)

Sv	91Apr	91May	91June	91Jul	91Aug	91Sep	91Oct	91Nov
2-17	1.618	0.554	0.474	1.100	2.092	0.709	1.506	3.128
3-17	3.056	2.560	2.110	2.651	2.934	2.643	2.437	2.234
6-17	1.811	0.386	0.708	1.669	2.536	0.771	1.494	2.943
11-17	5.108	3.888	3.471	4.276	6.178	5.409	5.253	6.136
12-17	2.146	2.578	2.499	2.289	1.669	2.309	1.475	2.084
13-17	3.460	2.878	1.940	3.186	4.082	1.682	1.951	3.666
14-17	1.448	-0.290	-0.665	0.539	1.156	-1.166	1.022	2.238
15-17	1.153	0.517	0.267	0.733	2.642	1.959	2.255	2.847
16-17	1.874	1.336	0.834	1.343	1.635	1.075	0.731	1.634
18-17	0.352	-0.361	-1.608	-0.862	1.137	1.199	1.200	2.174
19-17	2.527	1.058	0.915	1.751	3.820	2.928	3.097	4.083
20-17	-2.563	-1.713	-1.537	-1.591	-2.506	-1.008	-0.983	-1.337
21-17	-2.850	-1.487	-1.291	-1.359	-1.413	-0.765	-0.708	0.147
23-17	-1.599	-1.033	-0.667	-1.413	-1.027	-0.513	-0.687	-0.573
24-17								
17	-0.032	-0.151	-0.224	-0.826	-1.548	-1.363	-1.433	-1.998

Sv	91Dec	92Jan	92 Feb	92 Mar	92 Apr	92 May	Avg	Sig
2-17	1.931	1.908	0.704	1.630	1.326	0.776	1.390	0.714
3-17	2.018	2.700	2.582	3.092	2.594	2.192	2.557	0.323
6-17	2.043	1.168	0.164	1.694	1.539	0.996	1.423	0.760
11-17	5.181	4.347	3.132	4.110	5.832	4.534	4.775	0.924
12-17	3.132	2.321	2.597	2.076	2.502	2.518	2.300	0.395
13-17	3.939	3.568	2.608	2.657	4.256	3.412	3.092	0.799
14-17	1.002	0.890	-1.269	-1.265	-0.522	-0.805	0.165	1.109
15-17	1.859	1.650	-0.257	0.867	1.454	0.847	1.342	0.879
16-17	1.147	2.520	1.451	1.646	1.757	0.954	1.424	0.456
18-17	0.742	-0.069	-0.386	0.013	1.420	-0.249	0.336	0.989
19-17	3.045	2.337	0.687	2.176	3.107	1.679	2.372	1.017
20-17	-1.744	-2.304	-3.617	-2.730	-1.138	-0.695	-1.819	0.792
21-17	0.065	-1.495	-3.046	-2.014	-1.535	-0.829	-1.327	0.883
23-17	-0.421	-1.589	-2.108	-1.829	-0.948	-0.591	-1.071	0.525
24-17					1.500	0.774	1.137	0.363
17	-0.314	-0.194	-0.675	-1.323	-1.619	-0.783	-0.892	0.627

TABLE 4
Solution For Sv Bias
(L1-L2 in nanoseconds)

Sv	91Apr	91May	91June	91Jul	91Aug	91Sep	91Oct	91Nov
2	1.586	0.403	0.250	0.274	0.544	-0.654	0.073	1.130
3	3.024	2.409	1.886	1.825	1.386	1.280	1.004	0.236
6	1.779	0.235	0.484	0.843	0.988	-0.592	0.061	0.945
11	5.076	3.737	3.247	3.450	4.630	4.046	3.820	4.138
12	2.114	2.427	2.275	1.463	0.212	0.946	0.042	0.086
13	3.428	2.727	1.716	2.360	2.534	0.319	0.518	1.668
14	1.416	-0.441	-0.889	-0.287	-0.392	-2.529	-0.411	0.240
15	1.121	0.366	0.043	-0.093	1.094	0.596	0.822	0.849
16	1.842	1.185	0.610	0.517	0.087	-0.288	-0.702	-0.364
18	0.320	-0.512	-1.832	-1.688	-0.411	-0.164	-0.233	0.176
19	2.495	0.907	0.691	0.925	2.272	1.565	1.664	2.085
20	-2.595	-1.864	-1.761	-2.417	-4.054	-2.371	-2.416	-3.335
21	-2.882	-1.638	-1.515	-2.185	-2.961	-2.128	-2.141	-1.851
23	-1.631	-1.184	-0.891	-2.239	-2.575	-1.876	-2.120	-2.571
24								
17	-0.032	-0.151	-0.224	-0.826	-1.548	-1.363	-1.433	-1.998

Sv	91Dec	92Jan	92 Feb	92 Mar	92 Apr	92 May	Avg	Sig
2	1.617	1.714	0.029	0.307	-0.293	-0.007	0.498	0.711
3	1.704	2.506	1.907	1.769	0.975	1.409	1.666	0.681
6	1.729	0.974	-0.511	0.371	-0.081	0.213	0.531	0.698
11	4.867	4.153	2.457	2.787	4.213	3.751	3.884	0.712
12	2.818	2.127	1.922	0.753	0.883	1.735	1.408	0.901
13	3.625	3.374	1.933	1.334	2.637	2.629	2.200	0.979
14	0.688	0.696	-1.944	-2.588	-2.141	-1.588	-0.726	1.225
15	1.545	1.456	-0.932	-0.456	-0.166	0.064	0.451	0.713
16	0.833	2.326	0.776	0.323	0.138	0.171	0.532	0.804
18	0.428	-0.263	-1.061	-1.310	-0.199	-1.032	-0.556	0.693
19	2.731	2.143	0.012	0.853	1.487	0.896	1.480	0.766
20	-2.058	-2.498	-4.292	-4.053	-2.758	-1.478	-2.711	0.862
21	-0.249	-1.689	-3.721	-3.337	-3.155	-1.612	-2.219	0.882
23	-0.735	-1.783	-2.783	-3.152	-2.567	-1.374	-1.963	0.710
24					-0.119	-0.009	-0.064	0.055
17	-0.314	-0.194	-0.675	-1.323	-1.619	-0.783	-0.892	0.627

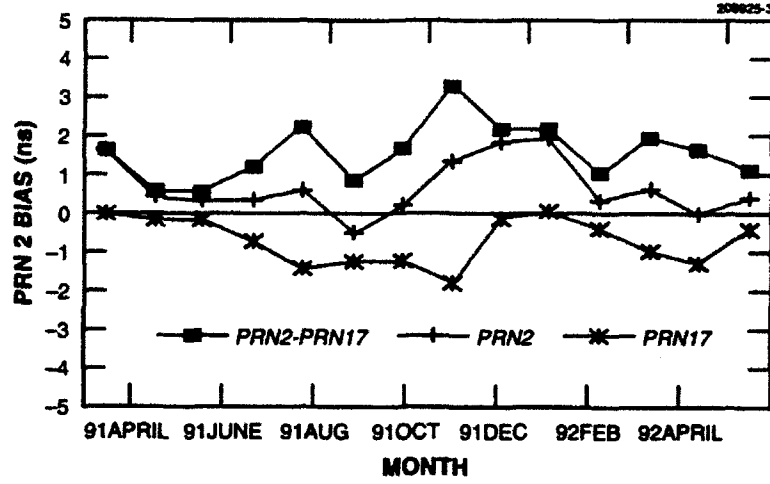


Figure 2. PRN 2 L1-L2 bias (nanoseconds): April 91 — May 92.

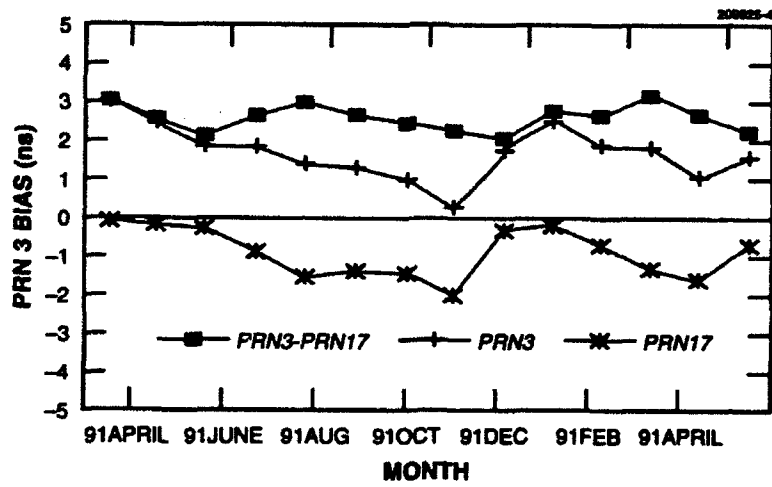


Figure 3. PRN 3 L1-L2 bias (nanoseconds): April 91 — May 92.

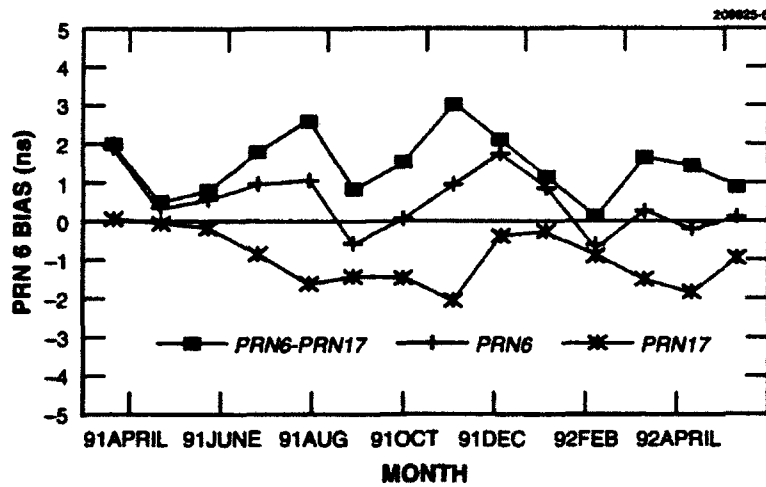


Figure 4. PRN 6 L1-L2 bias (nanoseconds): April 91 — May 92.

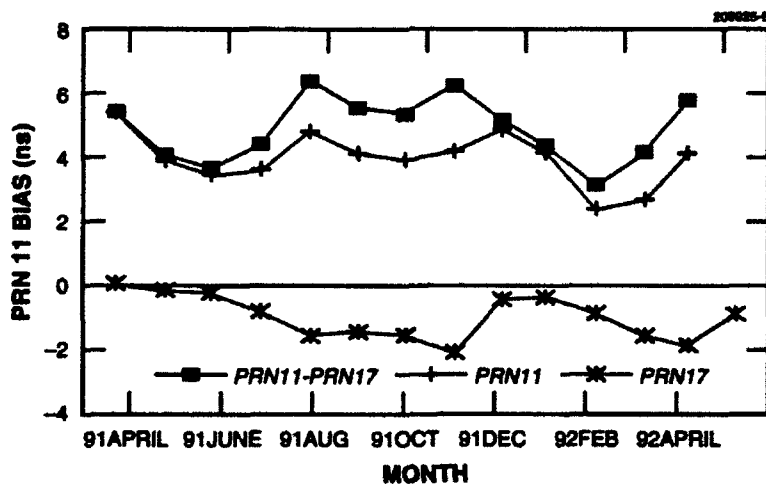


Figure 5. PRN 11 L1-L2 bias (nanoseconds): April 91 — May 92.

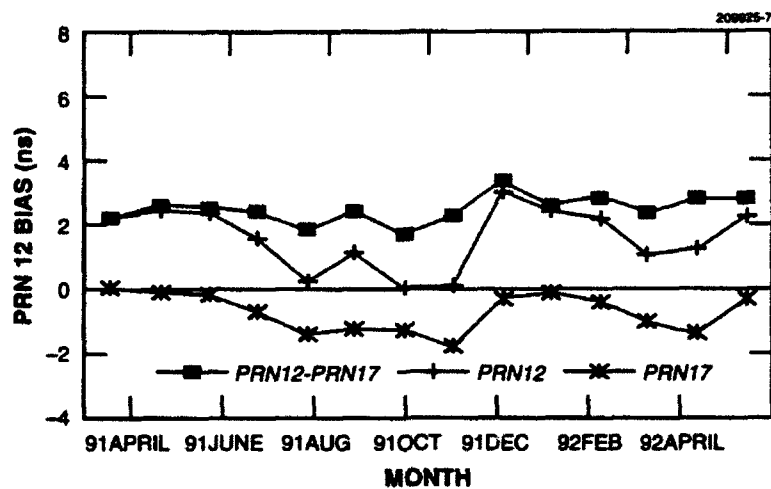


Figure 6. PRN 12 L1-L2 bias (nanoseconds): April 91 — May 92.

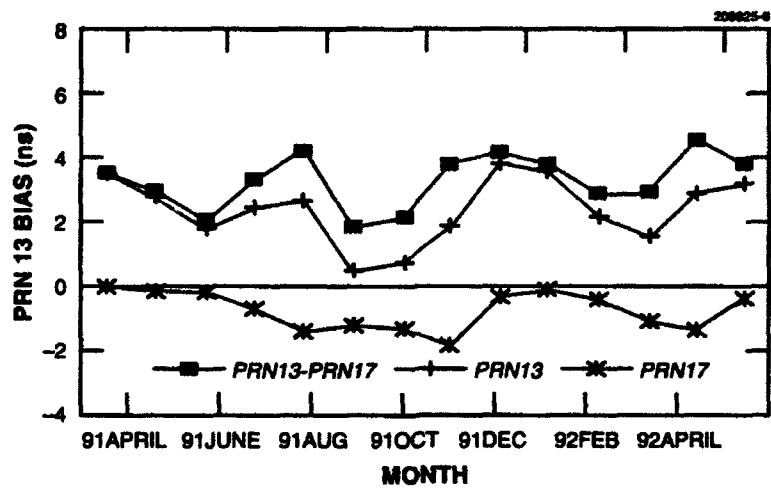


Figure 7. PRN 13 L1-L2 bias (nanoseconds): April 91 — May 92.

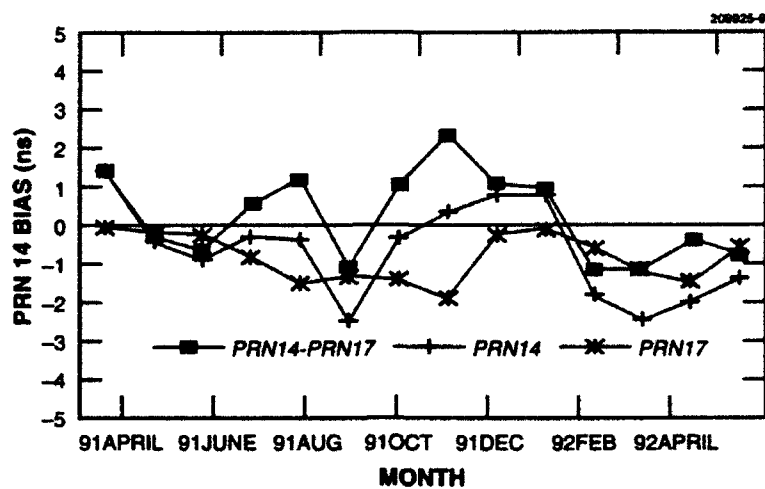


Figure 8. PRN 14 L1-L2 bias (nanoseconds): April 91 — May 92.

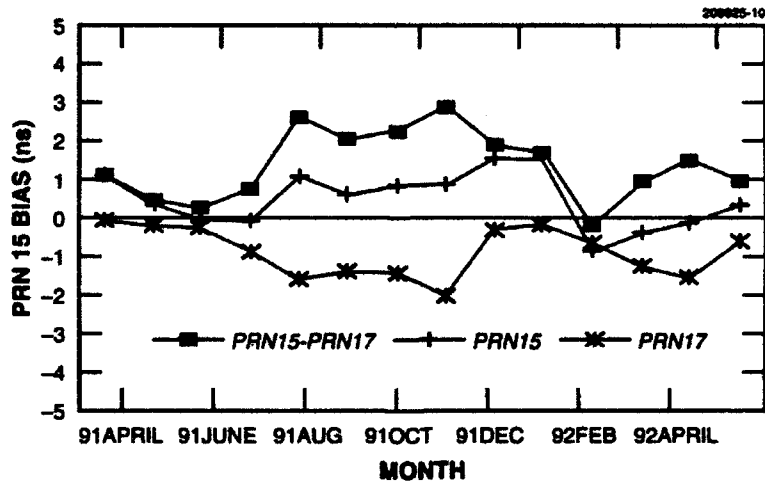


Figure 9. PRN 15 L1-L2 bias (nanoseconds): April 91 — May 92.

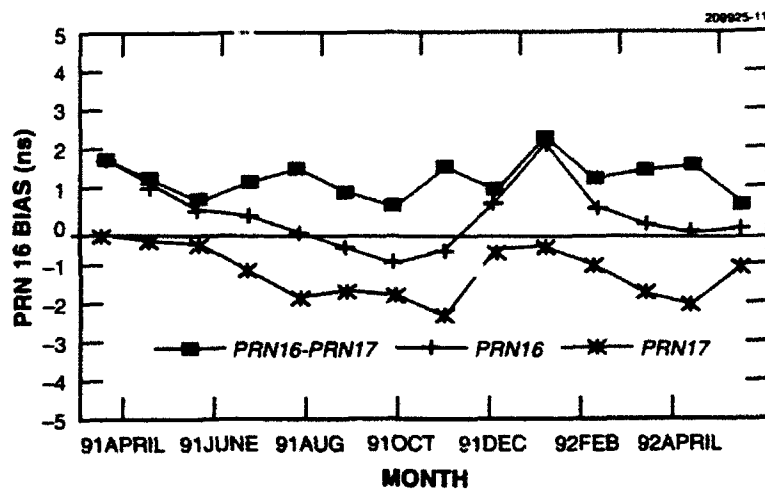


Figure 10. PRN 16 L1-L2 bias (nanoseconds): April 91 — May 92.

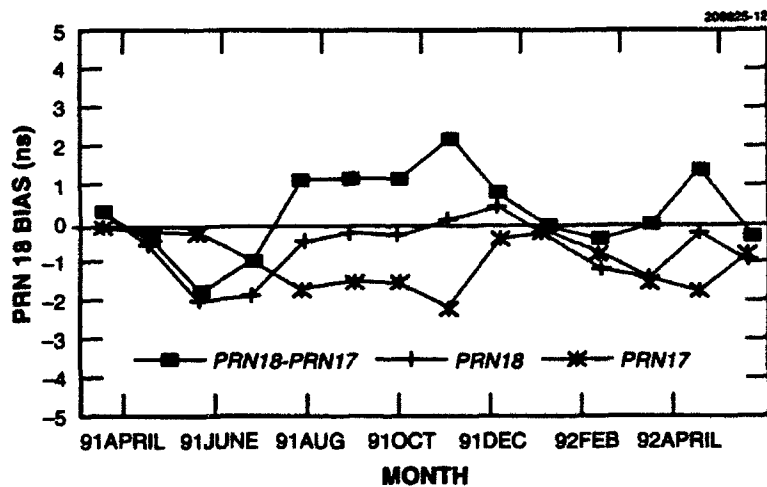


Figure 11. PRN 18 L1-L2 bias (nanoseconds): April 91 — May 92.

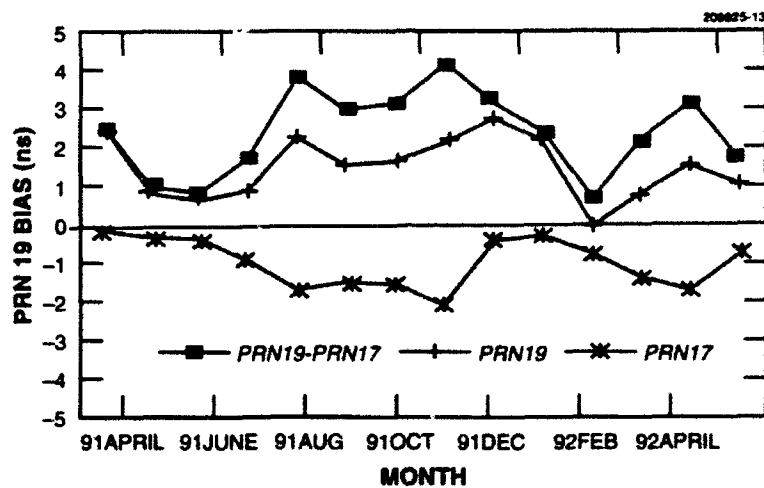


Figure 12. PRN 19 L1-L2 bias (nanoseconds): April 91 — May 92.

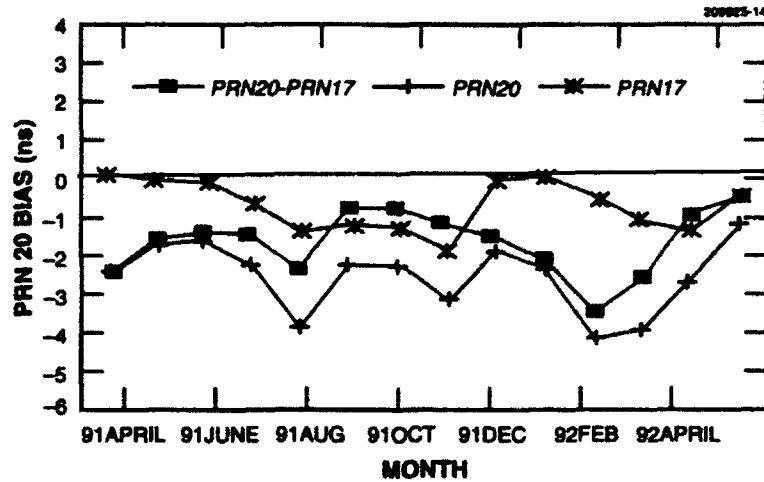


Figure 13. PRN 20 L1-L2 bias (nanoseconds): April 91 — May 92.

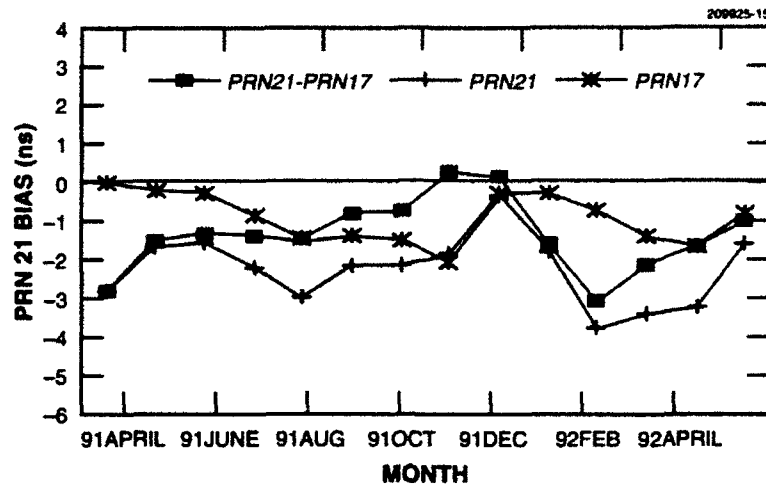


Figure 14. PRN 21 L1-L2 bias (nanoseconds): April 91 — May 92.

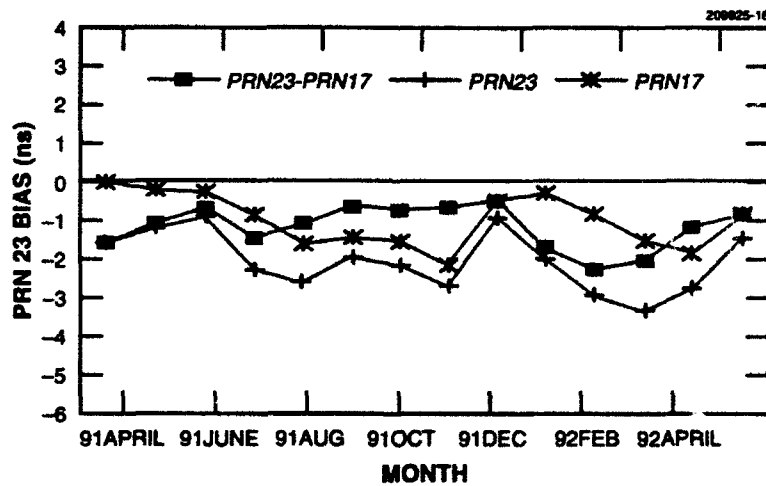


Figure 15. PRN 23 L1-L2 bias (nanoseconds): April 91 — May 92.

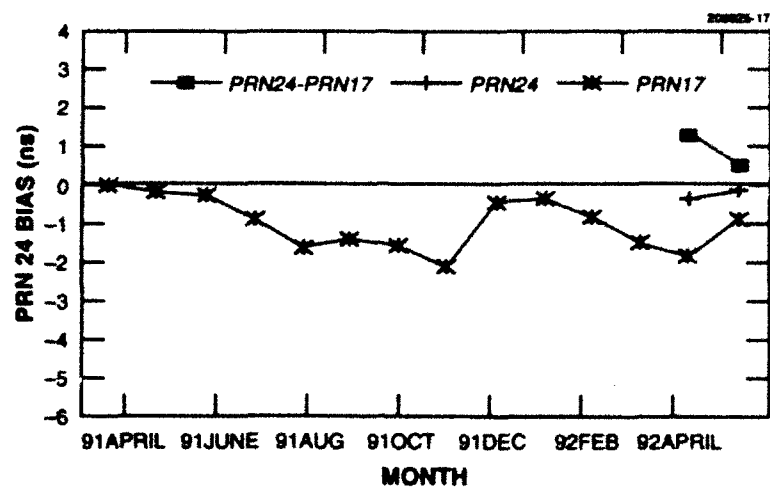


Figure 16. PRN 24 L1-L2 bias (nanoseconds): April 91 — May 92.

4. DISCUSSION

Given the formal 0.1-ns uncertainty of each bias determination, it is apparent that neither the difference in Sv bias difference between satellites nor the Sv bias for each satellite is constant. Some of the biases exhibit less variation than others. For example, Sv 17 has significant variation (i.e., its bias is clearly not constant), yet it seems to be one of the more stable satellites as evidenced by the standard deviation about the mean value. However, the Sv 17 solution could also include a real variation of receiver bias.

For a real-time system, some estimate of the bias is necessary to analyze current data. These bias values seem to change slowly. Therefore, one could use the most recent determination as the best estimate of "today's" value. Alternatively, one could use the average with some expected, but quantified, degradation of performance.

Table 5 lists a number of solutions for the Sv bias. The first column gives the Sv PRN number. Column 2 is a list of preflight calibration data [3]. We assume that the L2-L1 bias was reported in the Coker study [3], and we have changed the sign for the entries in this table. Similarly, column 3 is another list of biases [11]. Column 4 is a list of the biases used in the Mission Control Center (MCS) and is the basis of the bias broadcast in the GPS message. The MCS values were provided by J. Klobuchar. As called for by the ICD-GPS-200 paragraph 20.3.3.3.2, they are reported as $T_{gd}(\text{transmitted}) = -1.546(L1-L2)$, and the values given here have been obtained using this expression. The values for ARL and JPL (columns 5 through 7) were obtained from Coker [3], with the same sign reversal. The values for JPL91 (column 8), including the sign convention, were confirmed from an independent communication from JPL to J. Klobuchar [12]. Also included are a solution for the biases from Phillips Laboratory [PL91 (column 9)] and the mean bias values obtained in our analysis [LL92 (column 10)].

There is a disconcerting lack of agreement among the various candidate values. If there is truly a variation of the bias, then some of this disagreement could simply be due to the fact that the different numbers in Table 5 represent the bias at different times. If we assume the standard deviation about the mean represents the true statistics of each Sv bias, we could then inquire if any of these other bias values are a member of the same population. This inquiry cannot be done with the data in hand because there is not sufficient information about the other analyses. We can select values that fall within, say, 0.5 ns (≈ 1 sigma) of our mean value. There are 5 out of the 12 PL91 values that satisfy this criteria. There are 4 out of the 15 JPL91 values that satisfy this criteria. None of the 6 ARL values and only 3 out of the 13 prelaunch values satisfy this criteria. Increasing the allowed variation would improve this comparison.

The disagreement of all the analyses is of some concern. Some corollary results are available. The Millstone Hill GPS system is used for synoptically monitoring the ionosphere to provide a real-time ionosphere correction for radar-tracking data. Calibration data [1] have shown significant improvement in the ionosphere correction, as illustrated in Figure 17 taken from that work. The data in this figure represent the average bias in the range measured by the Millstone radar to the Lageos satellite. Lageos is tracked by NASA laser ranging stations, and the position of the Lageos satellite is known to within 30 cm. Therefore, the range error plotted in Figure 17 is either an error in Millstone's radar calibration or an error in the ionosphere correction. What is significant in this figure is the definite improvement

in smoothness of the average range biases after day 100. It was on this day that previous monthly GPS Sv biases, determined by our analysis, were correctly applied to the satellite data. We conclude that the radar system is sensitive to the correct Sv bias values, and that the system is at least internally consistent.

The Phillips Laboratory (PL) results, summarized in Table 5, were provided and plotted in Figures 18 through 22. The PL analysis, though based on similar principles, is quite different in terms of the GPS receiver, data selection, and data analysis. What is observed is a general similarity in temporal variation and a significant systematic difference. The PL91 results are generally 1.0 ns larger than the LL92 results. However, they both exhibit significant temporal variation of the Sv bias.

TABLE 5
L1-L2 Sv Bias (nanoseconds)

1	2	3	4	5	6	7	8	9	10
P R	C.C.	J.K.	MCS	JPL 86	ARL 87	ARL 89	JPL 91	PL91	LL 92
2	-1.1	-0.6	-0.6				0.2	-3.5	0.50
3	0.2		2.8		-1.1	-1.1	-1.2	3.1	1.67
6		2.3	1.3	1.8		-0.3	0.1	-2.7	0.53
11	-3.1	-3.3	0.1	-1.5	-3.2	-3.2	-3.0	4.3	3.88
12	0.0	0.0	-1.8		-3.0	-3.0	-3.0	1.8	1.41
13	-1.6	-1.8	-1.7		-1.8	-2.4	-1.6	0.1	2.20
14	-0.9	0.9	-0.9				0.5		-0.73
15	-1.9		-1.9				-0.2		0.45
16	-3.4	-3.6	-3.3				-0.4	1.5	0.53
17	-0.8	-0.8	-0.8				-1.0	-0.6	-0.89
18	1.3	1.3	1.3				3.4	-0.8	-0.56
19	-2.8	-2.8	-2.8				0.3	0.8	1.48
20	-2.5	-2.5	-2.5				1.0		-2.71
21	-3.6	-3.6	-4.0				-1.2	0.8	-2.22
23			-0.9				-2.5	-1.5	-1.96
24			-4.0						-0.06

After a discussion with PL [12], a series of detailed instrument comparisons were initiated. PL has two modern receivers: one developed at NIST that was used to obtain the results given in Table 5, and a second provided by Osborne Associates. PL provided a number of raw pseudorange measurements made simultaneously with the Millstone Hill receiver. The comparison of the simultaneous observation of L1-L2 at the two sites, separated by about 30 km, is shown in Figure 23. In this case we have used the carrier phase to smooth the pseudorange. More than 20 passes compared in this way revealed a constant offset $(L1-L2)_{MH} - (L1-L2)_{PL}$ ranging from -0.15 to $+0.50$ ns. These offsets could originate in one or both of the receivers. A number of passes exhibited a difference that changed nearly linearly. These comparisons are continuing. Nonzero multipath error is another possible source of these offsets. The receivers are in different locations with different multipath environments. It would be instructive to analyze simultaneous data from the two receivers using the same antenna.

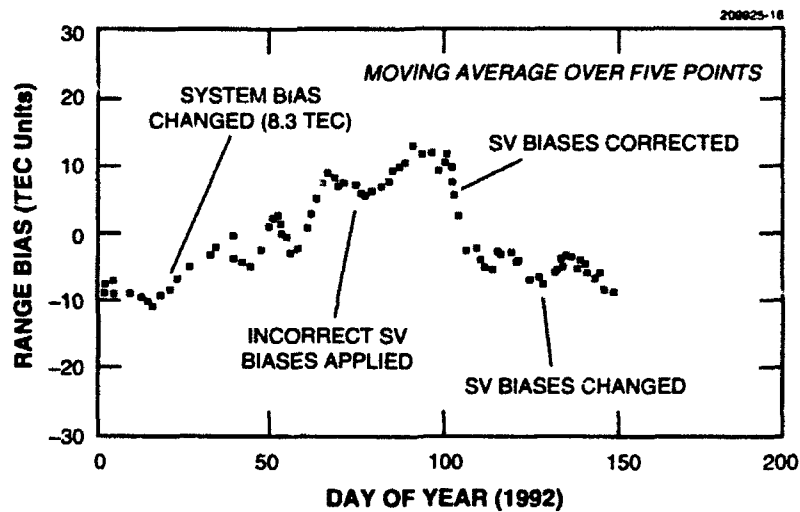


Figure 17. Lageos residuals using the GPS model.

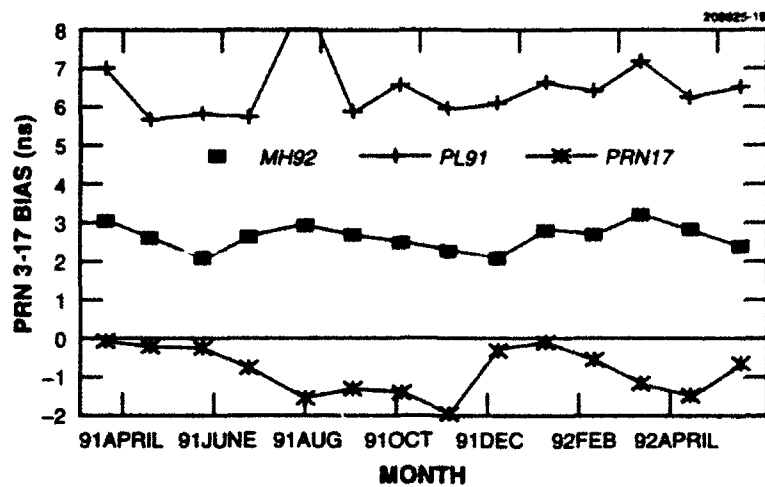


Figure 18. PRN 3-17 L1-L2 bias (nanoseconds): April 91 — May 92.

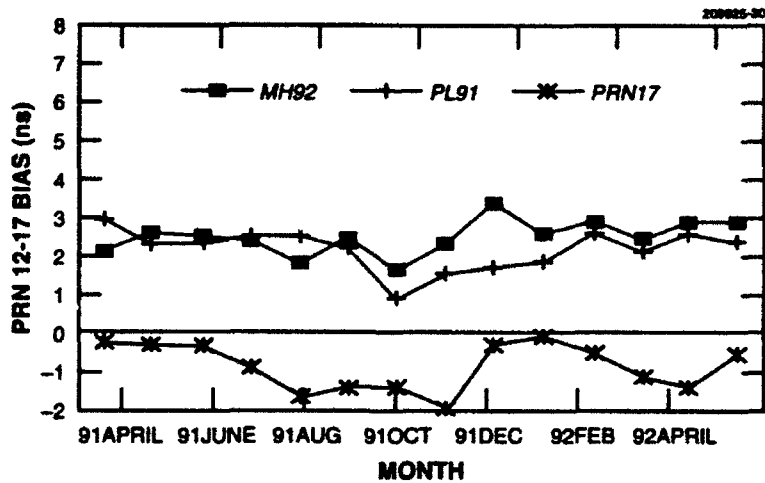


Figure 19. PRN 12-17 L1-L2 bias (nanoseconds): April 91 — May 92.

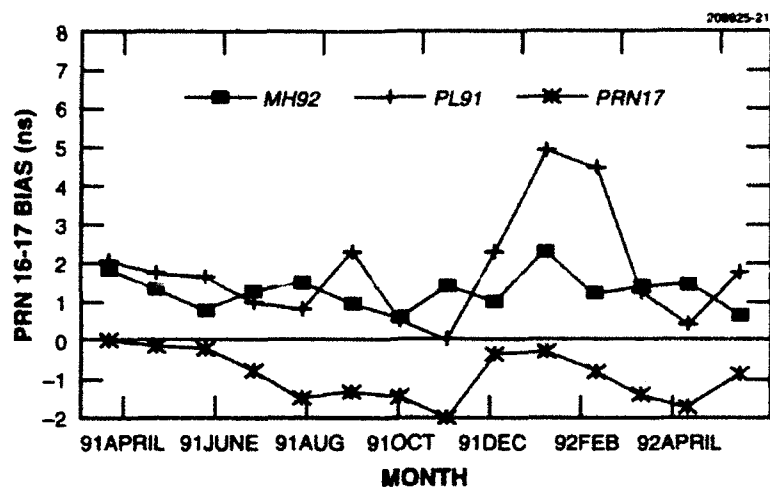


Figure 20. PRN 16-17 L1-L2 bias (nanoseconds): April 91 — May 92.

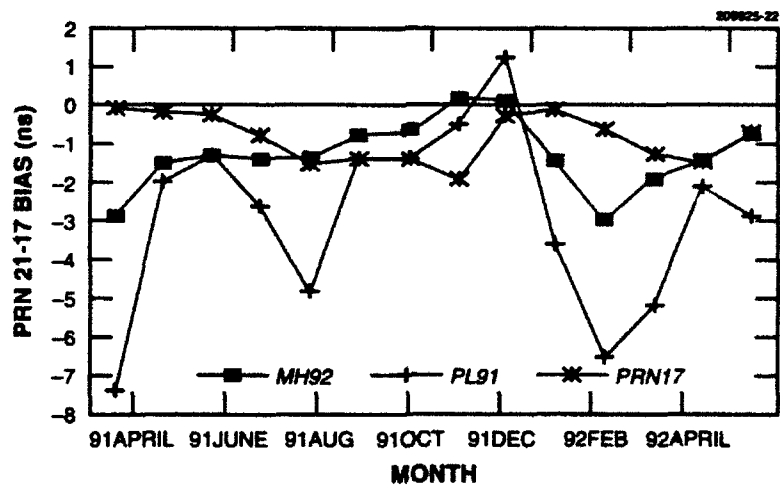


Figure 21. PRN 21-17 L1-L2 bias (nanoseconds): April 91 — May 92.

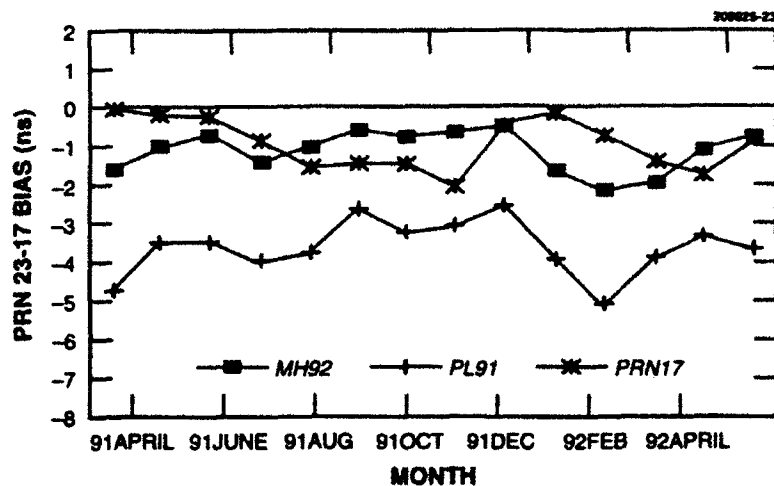


Figure 22. PRN 23-17 L1-L2 bias (nanoseconds): April 91 — May 92.

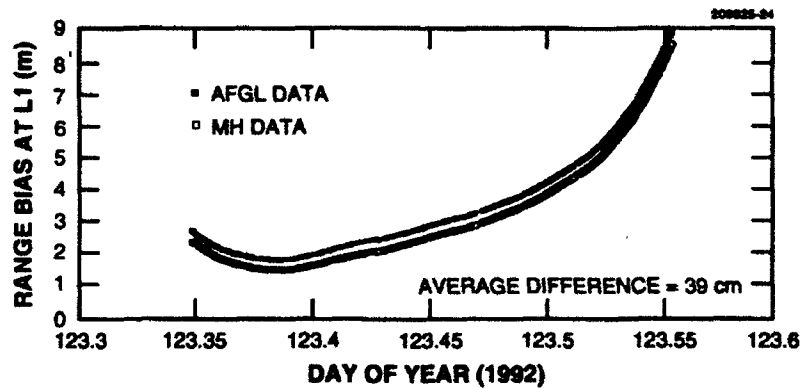


Figure 23. Sv biases - SSC# 20724, PRN 21 carrier smoothed pseudorange.

5. CONCLUSIONS

Monthly values for the GPS L1-L2 biases are recovered by differencing contemporaneous observations. The statistical accuracy of the bias recovery is about 0.1 ns (0.285 TEC units).

Real temporal variations in the space vehicle bias are observed. Additional data will be necessary to determine if the bias has a long-term trend or the observed statistical variability.

Significant variation occurs in the Sv bias. The Sv biases should be applied to any GPS data analysis that intends to monitor the ionosphere or remove the ionosphere path delay. For an a posteriori analysis, values obtained from such an analysis as this should be used.

REFERENCES

1. A.J. Coster, E.M. Gaposchkin, and L.E. Thornton, "Real-time ionospheric monitoring system using the GPS," MIT Lincoln Laboratory, Lexington, Mass., Technical Rep. 954 (24 August 1992).
2. J.A. Klobuchar (private communication, 1991).
3. C. Coker, "Satellite L1/L2 biases for GPS satellites," Applied Research Laboratory Memorandum No. CC110815 (15 August 1991).
4. R.S. Dahlke, D.S. Coco, and C.E. Coker, "Effect of GPS system biases on differential group delay measurements," ARL-TP-88-17 (August 1988).
5. G.E. Lanyi, T. Roth, and R.E. Neilan, "A comparison of mapped and measured total ionospheric electron content using GPS and beacon satellite observations," in J.M. Goodman, J.A. Klobuchar, R.G. Joiner, and H. Soicher (eds.) *Effect of the Ionosphere on Communications, Navigation, and Surveillance Systems* p. 135 (1987).
6. A.J. Coster (private communication, 1991).
7. D.S. Coco, C.E. Coker, S.R. Dahlke, and J.R. Clynych, "Variability of the GPS satellite differential group delay biases," *IEEE Trans. Aerosp. Electron. Syst.* 27, 6, 71-78 (1991).
8. A.J. Coster and E.M. Gaposchkin, "Use of GPS pseudo-range and phase data for measurement of ionospheric and tropospheric refraction," *Proc. of ION GPS-89*, Colorado Springs, Colorado (1989) 439-443.
9. A.J. Coster, E.M. Gaposchkin, L.E. Thornton, M.J. Buonsanto, and D. Tetenbaum, "Comparison of GPS and incoherent scatter measurements of the total electron content," in J.M. Goodman (ed.) *The Effect of the Ionosphere on Radiowave Signals and System Performance*, 460-469, based on Ionospheric Effect Symposium (1-3 May 1990).
10. J. Sciegieny (private communication, 1991).
11. J.A. Klobuchar (private communication, undated).
12. J.A. Klobuchar (private communication, 1992).

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